FLUX CHOICES FOR PARABOLIC/ELLIPTIC PROBLEMS: ALGORITHMS AND APPLICATIONS

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Although the original thrust of most discontinuous Galerkin research was in solving hyperbolic problems, the general proliferation of the DG methodology has also spread to the study of parabolic and elliptic problems. For example, works such as [3], in which the viscous compressible Navier-Stokes equations were solved, required that a discontinuous Galerkin formulation be extended beyond the hyperbolic advection terms to the viscous terms of the Navier-Stokes equations. Concurrently, both in [4] and [5] other discontinuous Galerkin formulations for parabolic and elliptic problems were proposed. In an effort to classify all the efforts made toward the use of DG methods for elliptic problems, Arnold et al., first in [1] and then more fully in [2], published a unified analysis of discontinuous Galerkin methods for elliptic problems.

In [2] a mathematical framework is provided for studying a variety of the different discontinuous Galerkin approaches for elliptic problems. In an attempt to ascertain which formulation was appropriate for us to use in the solution of the viscous compressible Navier-Stokes equations, we set out to study several of the different formulations presented in [2]. The three primary fluxes which we have examined are the Bassi-Rebay flux [3], the Baumann-Oden flux [4] and the LDG flux [5]

We present comparisons of the three different formulations in terms of both accuracy and stability. Specifically, we present studies of the solution of both the diffusion equation and the viscous Burgers equation for determining the 'h' (increasing number of element) and 'p' (increasing polynomial order per element) convergence and stability properties of these three fluxes. Our particular interest is when middle to high polynomial orders (polynomial orders of 5 to 15) are employed. We then examine the addition of stabilization parameters to these three fluxes, and will present numerical examples of the ramifications of these additions on both stability and accuracy. Lastly, we examine the practical implementation and parallelization issues involved in the use of these three fluxes in solving the viscous compressible Navier-Stokes equations.

References

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